SF 2017 – Abstract – Melcher

Title: Singularity formation in harmonic flows - two fundamental problems

Abstract: The formation of finite time singularities is a nonlinear phenomenon, which is attributed to a large class of partial differential equations as a break down of classical solvability. A recent trend in mathematical analysis is examine singularities themselves and trying to use

them as a means to tackle problems in other branches of mathematics (e.g. Poincare conjecture in topology) or to describe singular processes in science (switching topological states of matter). The project focusses on the harmonic map heat flow, a prototype of a singularity forming evolution equations related to various nonlinear field theories in mathematical physics. While most of the theory is developed in the framework of nonlinear PDE methods, we aim to promote a recent approach rather based on methods from geometric measure theory. Our newly developed concept for gradient flow for Cartesian currents is specifically tailored towards topological singularities of such flows. We aim to examine the following problems.

In the three dimensional case we ask whether a point singularity (hedgehog) emerging a a stable stationary solution is stable with respect to

the dynamic equations. The problem is outstanding and amounts to a substantial improvement of well-established partial regularity results.In the two dimensional case where stationary solutions are smooth we aim to establish conditions that guarantee the occurrence of finite time singularities and a resulting topological change in a controlled fashion. The problem is fundamental in various application based on the manipulation of topological states